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# Theory of a gyromagnetic Fabry–Pérot resonator

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**Abstract.** We discuss the transmission of electromagnetic radiation through a Fabry–Pérot resonator in the microwave or far-infrared frequency ranges where the magneto-optic properties are governed by a gyromagnetic permeability tensor. We restrict our attention to normally incident radiation but consider both Faraday (magnetic field normal to plate) and Voigt (field parallel to plate) geometries for ferromagnets and antiferromagnets. The effect of partially reflecting mirrors is included. It is shown that provided one uses the polarization eigenmodes, circular for Faraday geometry and plane for Voigt geometry, the transmission and reflection can be expressed by general formulae in terms of single-interface reflection and transmission coefficients. The resonator can be regarded as a magnetic-field tunable polarizer and it is shown how inclusion of mirrors sharpens the properties.

## 1. Introduction

This paper is concerned with an optical problem that can be simply stated: we consider transmission of normally incident electromagnetic radiation through a Fabry–Pérot etalon made of a magnetic material. We investigate the way in which resonances in the magnetic permeability  $\mu(\omega)$  affect the transmission spectrum and the relevant frequency range is therefore microwave for ferromagnets and far infrared for antiferromagnets. Here we consider the linear régime without exchange and in a subsequent paper [1] exchange is included.

The early work on ferromagnetic resonance (FMR) [2] was mainly concerned with metals in which the metallic conductivity  $\sigma$  results in a short skin depth  $\delta$ . The effective wave number  $k \sim 1/\delta$  is therefore large enough for exchange effects, represented in lowest order by a term  $Dk^2$  in the equations of motion, to come into play. Typical measurements are of the field dependence of absorption at a fixed microwave frequency; the absorption shows a strong peak at the peak of  $\text{Im}(\mu)$ , the imaginary part of the rf permeability. In recent years, FMR has been applied to detailed study of exchange constants in layered systems [3] but it is not necessary to review this work here. In the present work we discuss only insulators so that exchange effects arise only when  $k \sim 1/L$ ,  $L$  being the layer thickness, is sufficiently large.

The formalism to be presented here applies equally to antiferromagnets and we therefore briefly mention relevant papers on antiferromagnetic resonance (AFMR) and related topics; a full review has been given recently [4]. Early work [5,6] was concerned with the position, in frequency or field, of the AFMR line. More recently, attention has been focused on optical properties associated with the antiferromagnetic resonance; the theory involves application of Maxwell's equations with the relevant expression for  $\mu(\omega)$ . Sanders *et al* [7] studied transmission of far-infrared (FIR) radiation from a number of molecular laser

sources through discs of FeF<sub>2</sub>. It may be noted in particular that their experimental and theoretical results for the field dependence of the transmission of 1.36 THz radiation through a 786 μm Mn<sup>2+</sup> doped FeF<sub>2</sub> disc show resonance features and Fabry–Pérot fringes of the type to be discussed here. In a later paper [8] concerned with the field dependence of microwave transmission through thin (0.23 and 0.98 μm) epitaxial MnF<sub>2</sub> films the same group was able to show standing-spin wave fringes arising from an exchange ( $Dk^2$ ) term.

The Fabry–Pérot interferometer is a basic optical device that is discussed in all optics texts. The incorporation of partially reflecting mirrors, however, means that its properties are far from simple. We believe that it is worthwhile to study the properties of a magnetic Fabry–Pérot interferometer particularly since either or both exchange and nonlinear effects may lead to observable effects in the transmission. We discuss here the linear theory without exchange, which is dealt with in a subsequent paper [1]. In view of the ability of a magnetic medium to modify the polarization of transmitted light, this paper may be regarded as complementary to theoretical studies [9–12] that have appeared on the linear response of a Fabry–Pérot interferometer in which the dielectric medium is optically active.

We should comment briefly on experimental techniques. Ferromagnetic resonances are located in the microwave region where sources are of fixed frequency and spectra are naturally presented as magnetic-field scans. Antiferromagnetic resonances are at higher frequencies, say sub-millimetre to FIR, examples being the resonances in MnF<sub>2</sub> at 8 cm<sup>-1</sup> (240 GHz) and in FeF<sub>2</sub> at 52 cm<sup>-1</sup> (1.56 THz). Intense single-frequency sources are available at the lower end of this range and studies of MnF<sub>2</sub> often involve field scans at fixed frequency. However, modern FIR Fourier-transform spectrometers are able to produce frequency-scan spectra down to about 10 cm<sup>-1</sup> with dielectric beam splitters or about 2 cm<sup>-1</sup> with wire-grid beam splitters [13]. As reviewed elsewhere [4], starting with the work of Häussler *et al* [14, 15] on FeF<sub>2</sub> and CoF<sub>2</sub>, a substantial number of frequency-scanned spectra have been published with resolution now down to about 0.02 cm<sup>-1</sup> [16].

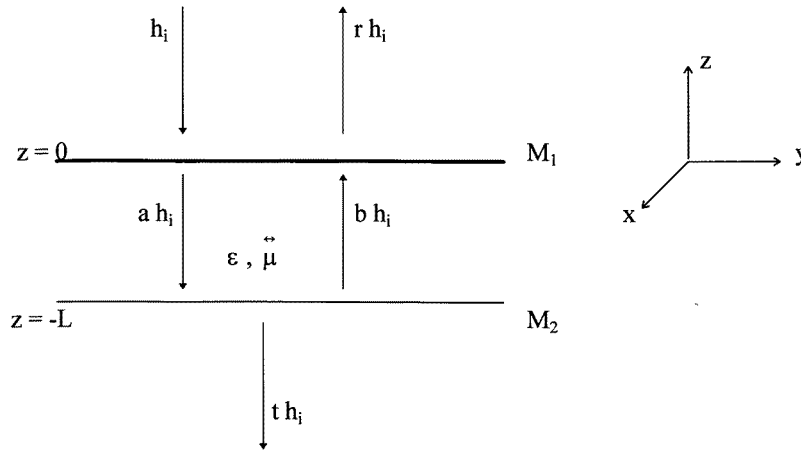
The formalism is developed in section 2 in which the transmission of the Fabry–Pérot interferometer is expressed in terms of the transmission and reflection coefficients of a single interface [17]. The results are illustrated numerically in section 3 for the ferromagnetic and antiferromagnetic insulators YIG and FeF<sub>2</sub> in the Faraday geometry (normal field) with brief comments on the Voigt geometry (parallel field) and conclusions are presented in section 4. A brief preliminary account of the part of this work that deals with the ferromagnet in the Faraday geometry without mirrors has been given previously [18].

## 2. Theory

The interferometer, axes and notation to be used are defined in figure 1. Electromagnetic radiation (microwave or infrared) is incident normally on a magnetic film of thickness  $L$  characterized, in this work, by a gyromagnetic permeability tensor  $\boldsymbol{\mu}(\omega)$  and dielectric constant  $\epsilon$ . The applied magnetic field may be either normal to the film, along  $z$  (Faraday geometry) or in plane (Voigt geometry). Partially reflecting mirrors M<sub>1</sub> and M<sub>2</sub> are included in the calculation. The rf magnetic field in the incident beam is written  $h_i$  and in a linear calculation the other beam magnitudes are proportional to  $h_i$ , as indicated.

We start with a ferromagnet in the Faraday geometry. The derivation of the linear magnetic susceptibility  $\chi(\omega)$  and the resulting permeability  $\boldsymbol{\mu} = \mathbf{I} + \boldsymbol{\chi}$  is standard [19] and  $\boldsymbol{\chi}$  is diagonalized by the transformations  $m^\pm = m_x \pm im_y$  and  $h^\pm = h_x \pm ih_y$ :  $m^\pm = \chi^\pm h^\pm$  with

$$\chi^\pm = \frac{\omega_m \pm i\Gamma}{(\omega_0 - \omega_m) \pm \omega \pm i\Gamma(\omega_0/\omega_m - 1)}. \quad (1)$$



**Figure 1.** Magnetic Fabry–Pérot interferometer with axes and notation to be used.

Here  $\Gamma$  is the coefficient of a Landau–Lifshitz damping term  $-\Gamma M \times (M \times H)/M_S^2$  in the equation of motion for  $M$ . The frequencies are defined by  $\omega_0 = \gamma H_0$  and  $\omega_m = \gamma M_S$ , where  $H_0$  and  $M_S$  are the static field and static magnetization, and demagnetization is included by the replacement  $\omega_0 \rightarrow \omega_0 - \omega_m$  compared with the standard form. It follows from (1) that the propagating eigenmodes are the two states of circular polarization. It will be convenient here to use the convention that the sign of the polarization is defined by the direction of rotation with respect to the applied magnetic field. It should be noticed that this is different from the standard optics definition, in which the sign is defined in terms of rotation looking towards the source. One advantage of the definition we are using is that at the interfaces  $+$  reflects to  $+$  and  $-$  to  $-$ . The positions of the resonances in  $\chi^+$  and  $\chi^-$  play an important part in the Fabry–Pérot response. Neglecting damping, they are at

$$\begin{aligned} \chi^- : \quad & \omega_0 = \omega + \omega_m \\ \chi^+ : \quad & \omega_0 = \omega_m - \omega \quad \omega < \omega_m \end{aligned}$$

for field sweep.

Since the sign of the circular polarization (as we are defining it) is preserved both on reflection and transmission at an interface and in propagation through the magnetic medium, the standard analysis for an isotropic medium may be applied as long as the results are applied for the eigenstates  $m^\pm$ . The most convenient formulation [17] is found by expressing the outgoing waves at each interface in terms of the incoming waves. Thus at the upper interface

$$a = \tau_{12} + \rho_{21}b \quad (2)$$

$$r = \rho_{12} + \tau_{21}b \quad (3)$$

where  $\tau_{ij}$  and  $\rho_{ij}$  are the complex amplitude transmission and reflection coefficients for radiation incident in medium  $i$  on the interface with medium  $j$ . Likewise at the lower interface

$$t = \tau_{23}a\delta \quad (4)$$

$$b\delta^{-1} = \rho_{23}a\delta \quad (5)$$

where  $\delta = \exp(ikL)$  is the phase shift in propagation across the film and  $k$  is the wave number given by  $k^2 = (1 + \chi)\varepsilon\omega^2/c^2$  where  $\varepsilon$  is the dielectric constant of the film. The reflection and transmission amplitudes as well as  $\delta$  and  $k$  all carry superscript + or -, left out of the above equations for ease of notation. Solution of (2)–(5) is straightforward and leads to

$$t = \frac{\tau_{12}\tau_{23}\delta}{1 - \rho_{21}\rho_{23}\delta^2} \quad (6)$$

$$r = \rho_{12} + \frac{\tau_{12}\tau_{21}\rho_{23}\delta^2}{1 - \rho_{21}\rho_{23}\delta^2} \quad (7)$$

with superscripts  $\pm$  implied on all quantities.

We now state expressions for the single-interface coefficients including the effect of a partial mirror. We represent the mirror as a metal of conductivity  $\sigma_M$  and negative dielectric constant  $\varepsilon_M$ , with thickness  $d$  much less than the optical wavelength so that the  $H$  boundary condition is  $\Delta H_{\parallel} = d(\sigma_M - i\omega\varepsilon_M)E_{\parallel}$ . It follows that

$$\tau_{ij} = \frac{2k_i}{k_i + k_j + \alpha} \quad (8)$$

$$\rho_{ij} = \frac{k_i - k_j - \alpha}{k_i + k_j + \alpha} \quad (9)$$

where  $\alpha = d(\sigma_M - i\omega\varepsilon_0\varepsilon_M)\omega\mu_0$  is the mirror term and  $k_i$  is the wave number. In numerical illustrations we take  $d \rightarrow 0$  and  $\sigma_M, \varepsilon_M \rightarrow \infty$  with the products finite and we generally choose  $\sigma_M d = 0$ , non-dissipative mirrors.

The above results also hold for the Voigt geometry,  $H_0$  in the plane of the film, say along  $y$  in figure 1. The bulk eigenmodes are then the plane polarizations, with  $h$  along  $x$  coupled to the magnetic resonances and  $h$  along  $y$  uncoupled. For the former, the wave number  $k_V$  in the medium is given by  $k_V^2 = \varepsilon\mu_V\omega^2/c^2$  where  $\mu_V$  is the Voigt permeability. With damping terms omitted for simplicity

$$\mu_V = \frac{(\omega_0 + \omega_m)^2 - \omega^2}{\omega_0^2 - \omega^2 + \omega_m\omega_0}. \quad (10)$$

With obvious substitutions of  $\mu_V$  and  $k_V$  the optical formalism continues to apply.

The formal results apply for antiferromagnets although it should be noted that in general the form of  $\boldsymbol{\mu}(\omega)$  then depends on the angle between the applied field and the ordering direction [20–22]. In the uniaxial antiferromagnets  $\text{MnF}_2$  and  $\text{FeF}_2$  ordering in the absence of a field is along the tetragonal  $c$  axis and for a static field applied in the same direction  $\mu^{\pm} = \mu_1 \mp \mu_2$ , with [23]

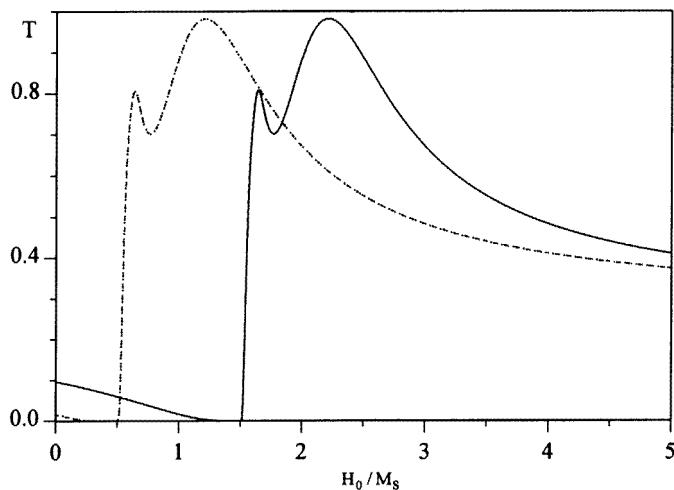
$$\mu_1 = 1 + \gamma^2 H_A M_S (Y^+ + Y^-) \quad (11)$$

$$\mu_2 = \gamma^2 H_A M_S (Y^+ - Y^-) \quad (12)$$

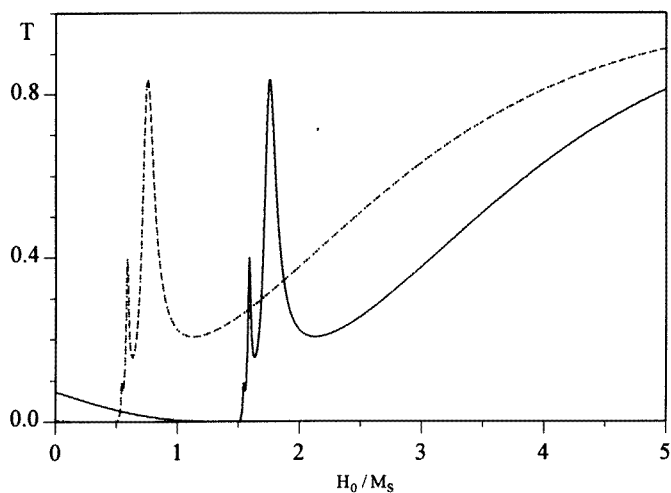
where  $H_A$  is the anisotropy field and  $M_S$  the equilibrium sublattice magnetization. The factors  $Y^+$  and  $Y^-$  are given by  $Y^{\pm} = [\omega_r^2 - (\omega \pm \gamma H_0)^2]^{-1}$  and  $\omega_r$  is the resonant frequency  $\gamma(2H_A H_E + H_A^2)^{1/2}$  where  $H_E$  is the exchange field. Because  $\boldsymbol{\mu}$  takes the standard form the results (6)–(9) apply first for  $c$  axis and field both normal to the mirrors and second for  $c$  axis and field both parallel to the mirrors. For the former case there is of course no correction for demagnetization since the net magnetization of the antiferromagnet is zero.

### 3. Numerical results and discussion

We illustrate our results with graphs of transmission coefficients  $T^\pm = |t^\pm|^2$  first for a 1 cm film of the insulating ferromagnet YIG and second for a 1 mm film of the insulating antiferromagnet FeF<sub>2</sub>; the external medium is taken as air. The material parameters for YIG are  $\varepsilon = 15.4$ ,  $M_S = 1750$  Gauss and we assume  $\Gamma = 0.01\gamma M_S$  while for FeF<sub>2</sub>  $\varepsilon = 5.5$ ,  $H_E = 53.3$  T,  $H_A = 19.7$  T and we take  $\Gamma = 10^{-4}\gamma M_S$ , as found in reflectivity from a good-quality sample [21, 22]. The mirrors are taken as non-absorbing ( $\sigma_M = 0$ ). To



(a)



(b)

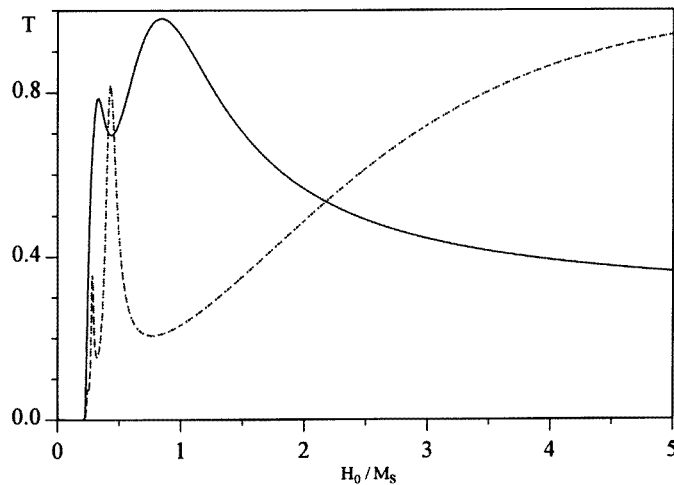
**Figure 2.** Magnetic-field dependence at frequency  $\omega = 0.5\omega_m$  of Faraday-geometry transmission coefficients  $T^-$  (—) and  $T^+$  (---) of a 1 cm YIG film with mirror reflectivity (a)  $R = 0.353$  and (b)  $R = 0.6$ .

characterize the mirror we quote a ‘reflectivity’  $R$  which is the single-interface reflectivity at the resonance frequency  $\omega_0$  or  $\omega_r$  due to the mirror and the dielectric constant alone, i.e. with  $\mu$  set equal to the unit tensor.

In order to make comparison with standard optical results we have computed graphs of transmission versus frequency for YIG in a perpendicular field [18] but the curves are not shown here. The main features are these. Fringes are present in both + and – polarization and they become much sharper when the mirror reflectivity  $R$  is increased. The transmission drops to zero in an interval around the resonance frequency because of resonant absorption. In all cases the transmission approaches unity at low frequency; this is the standard result for a film of thickness much less than the optical wavelength [24].

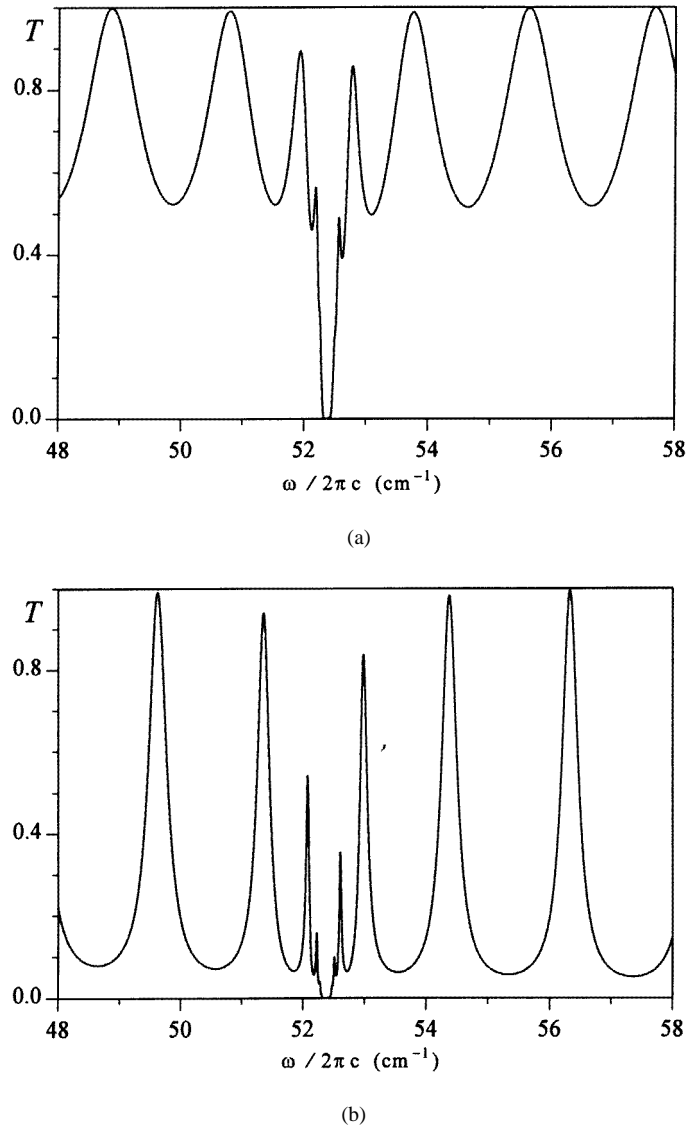
Calculated field-scan transmissivities for YIG in a perpendicular field are shown in figure 2 for frequency  $\omega = 0.5\omega_m$ . As noted in section 2, the resonant fields are  $0.5\omega_m$  for  $T^+$  and  $1.5\omega_m$  for  $T^-$ . A loss in transmission due to absorption in the resonant region in either case is clearly seen and away from resonance there are typical Fabry–Pérot fringes. Comparison of figures 2(a) and (b) shows the striking effect of the inclusion of mirrors in bringing out additional structure in the curves.

As mentioned, the formalism applies for a ferromagnet in the Voigt configuration (parallel field) and some calculated transmission curves for the plane polarization that couples to the magnetic resonance are shown in figure 3. For a field scan, it follows from (10) that the resonant field is given by  $\omega_0 = [(\omega_m^2 + 4\omega^2)^{1/2} - \omega_m]/2$  which for figure 3 gives  $H_0/M_S = 0.212$ . Resonant absorption means that the transmission is near zero around this field value. Away from resonance, it is seen that as in figure 2 the mirror reflectivity has a considerable influence on transmission.



**Figure 3.** Calculated transmission curves for a 1 cm YIG film in the Voigt configuration ( $H_0$  and  $M_S$  in plane with microwave  $h$  field transverse to  $M_S$ ). The curves are for field dependence at frequency  $\omega = 0.5\omega_m$  for mirror reflectivities  $R = 0.353$  (—) and  $R = 0.6$  (---).

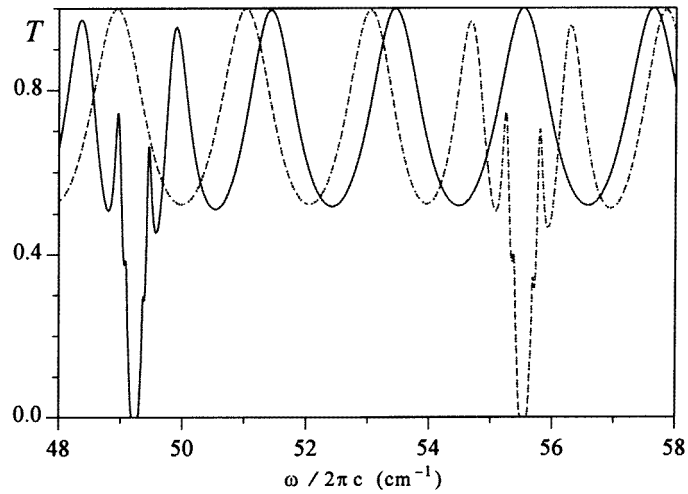
Finally, we show in figure 4 calculated transmission curves for  $\text{FeF}_2$  with the  $c$  axis and applied field normal to the mirrors. For this case we show frequency scans since, as remarked in section 1, for this part of the spectrum frequency scans are practical and a considerable number of data of this kind have already been published. From an illustrative point of view, moreover, frequency scans have the advantage of easier comparison with



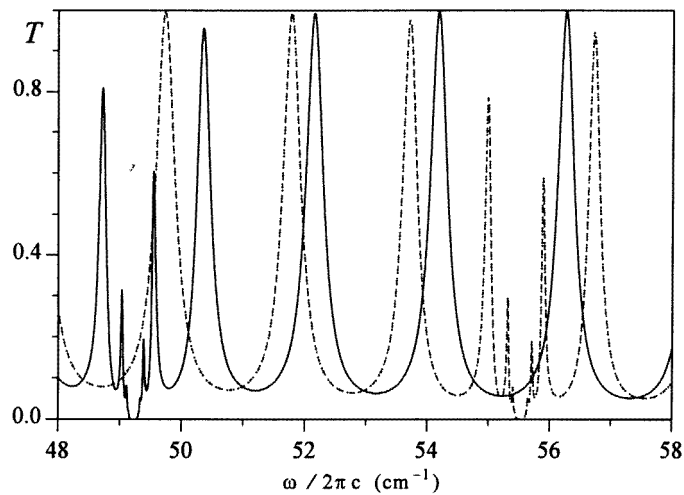
**Figure 4.** Calculated curves for frequency dependence of the transmission through a 1 mm  $\text{FeF}_2$  film with  $c$  axis and applied field  $H_0$  normal to the mirrors: (a) both  $T^+$  and  $T^-$  for  $H_0 = 0$  and no mirrors, (b) both  $T^+$  and  $T^-$  for  $H_0 = 0$  and  $R = 0.6$ , (c)  $T^+$  (—) and  $T^-$  (---) for  $H_0 = 3$  T and no mirrors, (d)  $T^+$  (—) and  $T^-$  (---) for  $H_0 = 3$  T and  $R = 0.6$ .

the optics literature. The reasons for the general structure of the curves in figure 4 are clear from well known properties of the permeability of (11) and (12). The resonances are very narrow because of the small value of the numerator  $\gamma^2 H_A M_S$ ; typically the resonant linewidth is of the order of  $0.1 \text{ cm}^{-1}$  or less [21]. The factors  $Y^+$  and  $Y^-$  are equal in zero field; consequently in that case the optical properties are isotropic with  $T^+$  and  $T^-$  equal, as shown in figure 4(a) and (b). The zero-field transmission drops to zero as a result of absorption in a narrow interval around resonance. Away from resonance figure 4(a)





(c)



(d)

**Figure 4.** (Continued)

and (b) shows typical Fabry–Pérot fringes resulting from optical standing waves given by  $\Delta k = \pi/L$ . The fringe spacings are fairly uniform but it can be seen that the spacing in frequency becomes closer near resonance since  $\Delta\omega = (d\omega/dk)\Delta k$  and  $d\omega/dk$  is relatively small on either side of resonance. Away from the narrow resonance region regular fringes are seen and comparison between figure 4(a) and (b) shows how these are sharpened by the inclusion of mirrors. Increasing  $R$  increases the finesse  $\mathcal{F} = \pi R^{1/2}/(1 - R)$  of the Fabry–Pérot etalon and away from resonance the comparison between figure 4(a) and (b) is similar to that found in many optics texts in a discussion of the importance of the finesse.

In non-zero field the difference between  $Y^+$  and  $Y^-$  is a standard Zeeman splitting as seen clearly in reflection experiments [21]. The transmissions  $T^+$  and  $T^-$  show the same splitting with behaviour in the separate resonance regions similar to that of the zero-field

curves. Although the width of the resonances themselves is narrow it is seen that substantial differences between  $T^+$  and  $T^-$  persist over a substantial frequency range, in fact of order  $20 \text{ cm}^{-1}$  on either side of the zero-field resonance. The spacing between transmission peaks is approximately proportional to  $1/L$  and the splitting between  $T^+$  and  $T^-$  is proportional to  $H_0$ . By suitable choice of  $L$  and  $H_0$ , therefore, it should be possible to ensure that the maxima of  $T^+$  coincide with the minima of  $T^-$ . With small damping the maximum transmission is very close to unity and the minimum is  $(1 + 4\mathcal{F}^2/\pi^2)^{-1}$ . The implication of figure 4(d), therefore, is that with moderate or large  $\mathcal{F}$  and suitable choice of  $L$  the antiferromagnetic Fabry–Pérot interferometer can serve as a circular polarizer, tunable by the magnetic field.

The formalism here applies equally to the antiferromagnet in the Voigt geometry with  $c$  axis and field in plane. We have not computed transmission curves for this case but since the eigenmodes are plane polarized it is to be expected that for this geometry the Fabry–Pérot interferometer is a field-tunable plane polarizer.

#### 4. Conclusions

We have given a full account of the linear properties of the gyromagnetic Fabry–Pérot resonator without exchange. For either the ferromagnet or the antiferromagnet with field normal to the mirrors the eigenmodes are the circular polarization states and we have therefore expressed the results in terms of these. For unpolarized incident radiation, figure 4 shows that the Fabry–Pérot interferometer can act as a tunable circular polarizer and the inclusion of partially reflecting mirrors can make the polarization nearly 100%.

As mentioned in section 1, Fabry–Pérot fringes of the kind discussed here were observed by Sanders *et al* [7] in normal-incidence transmission through a  $786 \mu\text{m}$   $\text{FeF}_2$  sample. They used an unpolarized incident beam of fixed frequency and they show magnetic-field fringes over a frequency range that is sufficiently wide for the  $T^+$  and  $T^-$  fringes to be identical except presumably in narrow intervals near resonance. Their theoretical discussion is essentially identical to ours.

Comparison may be made between our results and those of Lalov and Miteva [9] and Lalov and Georgieva [10] on the optically active Fabry–Pérot interferometer: the former paper is for cases when absorption may be neglected while the latter includes absorption. Like us, Lalov and Miteva concentrate on the polarization properties of the reflected and transmitted light and in particular they emphasize the existence of a reflected beam of opposite polarization to the incident beam. The second paper considers the modification of the results by non-resonant absorption. Both these papers deal with a general angle of incidence whereas we have restricted our discussion to normal incidence. It is known [25] that for general propagation direction in a bulk ferromagnet the propagating eigenmodes are elliptically polarized and we believe that our formalism could be extended to oblique incidence without too much difficulty provided the problem is handled in terms of the eigenmodes.

For both ferromagnets and antiferromagnets the equations of motion from which  $\mu(\omega)$  is derived may be extended to include terms in  $D \nabla^2 m$  arising from the exchange interaction. When these are included, the propagation equation for the bulk material becomes a quadratic in  $k^2$  with solutions  $k_0$  and  $k_s$ , say which at most frequencies are of a very different magnitude and correspond, loosely, to optical and spin waves. The extension of the formalism to include these spin waves will be given in a subsequent paper [1].

The nonlinear properties of the ordinary Fabry–Pérot interferometer have been a matter of considerable interest [26, 27]. The permeabilities that we have used are derived by

linearizing the equations of motion for the magnetization terms. It is possible, however, to retain nonlinear terms [28]. One of the motivations of the present work has been to survey the results of the linear theory so as to have a clear basis for discussion of the nonlinear magnetic Fabry–Pérot interferometer, which we believe is an accessible problem.

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